Chapter Seven

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Gravitation

7.1 INTRODUCTION

Early in our lives, we become aware of the tendency of all material objects to be attracted towards the earth. Anything thrown up falls down towards the earth, going uphill is lot more tiring than going downhill, raindrops from the clouds above fall towards the earth and there are many other such phenomena. Historically it was the Italian Physicist Galileo (1564-1642) who recognised the fact that all bodies, irrespective of their masses, are accelerated towards the earth with a constant acceleration. It is said that he made a public demonstration of this fact. To find the truth, he certainly did experiments with bodies rolling down inclined planes and arrived at a value of the acceleration due to gravity which is close to the more accurate value obtained later.

A seemingly unrelated phenomenon, observation of stars, planets and their motion has been the subject of attention in many countries since the earliest of times. Observations since early times recognised stars which appeared in the sky with positions unchanged year after year. The more interesting objects are the planets which seem to have regular motions against the background of stars. The earliest recorded model for planetary motions proposed by Ptolemy about 2000 years ago was a 'geocentric' model in which all celestial objects, stars, the sun and the planets, all revolved around the earth. The only motion that was thought to be possible for celestial objects was motion in a circle. Complicated schemes of motion were put forward by Ptolemy in order to describe the observed motion of the planets. The planets were described as moving in circles with the centre of the circles themselves moving in larger circles. Similar theories were also advanced by Indian astronomers some 400 years later. However a more elegant model in which the Sun was the centre around which the planets revolved – the 'heliocentric' model – was already mentioned by Aryabhatta (5^{th} century A.D.) in his treatise. A thousand years later, a Polish monk named Nicolas Copernicus (1473-1543) proposed a definitive model in which the planets moved in circles around a fixed central sun. His theory was discredited by the church, but notable amongst its supporters was Galileo who had to face prosecution from the state for his beliefs.

It was around the same time as Galileo, a nobleman called Tycho Brahe (1546-1601) hailing from Denmark, spent his entire lifetime recording observations of the planets with the naked eye. His compiled data were analysed later by his assistant Johannes Kepler (1571-1640). He could extract from the data three elegant laws that now go by the name of Kepler's laws. These laws were known to Newton and enabled him to make a great scientific leap in proposing his universal law of gravitation.

7.2 KEPLER'S LAWS

The three laws of Kepler can be stated as follows:

1. Law of orbits : All planets move in elliptical orbits with the Sun situated at one of the foci of the ellipse (Fig. 7.1a). This law was a deviation from the Copernican model which allowed only circular orbits. The ellipse, of which the circle is a special case, is a closed curve which can be drawn very simply as follows.

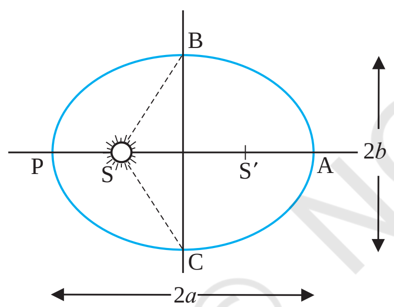


Fig. 7.1(a) *An ellipse traced out by a planet around the sun. The closest point is P and the farthest point is A, P is called the perihelion and A the aphelion. The semimajor axis is half the distance AP.*

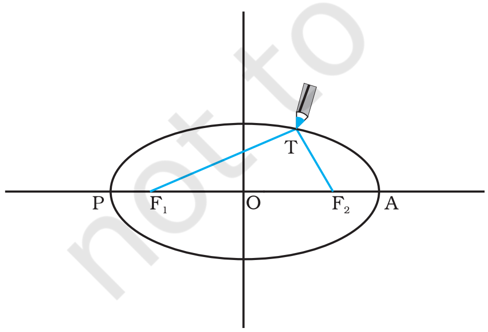


Fig. 7.1(b) *Drawing an ellipse. A string has its ends fixed at F_1 and F_2. The tip of a pencil holds the string taut and is moved around.*

Select two points F_1 and F_2. Take a length of a string and fix its ends at F_1 and F_2 by pins. With the tip of a pencil stretch the string taut and then draw a curve by moving the pencil keeping the string taut throughout.(Fig. 7.1(b)) The closed curve you get is called an ellipse. Clearly for any point T on the ellipse, the sum of the distances from F_1 and F_2 is a constant. F_1, F_2 are called the focii. Join the points F_1 and F_2 and extend the line to intersect the ellipse at points P and A as shown in Fig. 7.1(b). The midpoint of the line PA is the centre of the ellipse O and the length PO = AO is called the semi-major axis of the ellipse. For a circle, the two focii merge into one and the semi-major axis becomes the radius of the circle.

2. Law of areas : The line that joins any planet to the sun sweeps equal areas in equal intervals of time (Fig. 7.2). This law comes from the observations that planets appear to move slower when they are farther from the sun than when they are nearer.

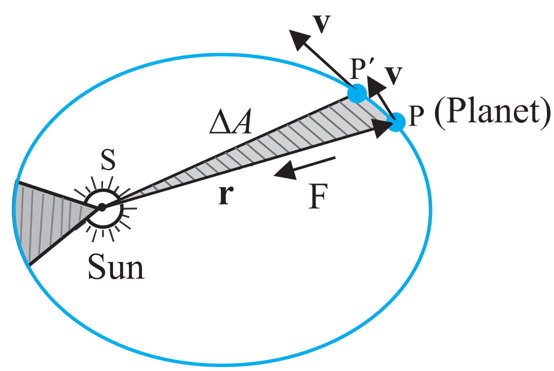


Fig. 7.2 *The planet P moves around the sun in an elliptical orbit. The shaded area is the area ΔA swept out in a small interval of time Δt.*

3. Law of periods : The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet.

Table 7.1 gives the approximate time periods of revolution of eight* planets around the sun along with values of their semi-major axes.

Table 7.1 Data from measurement of planetary motions given below confirm Kepler's Law of Periods

(a ≡ Semi-major axis in units of 10^{10} m.
T ≡ Time period of revolution of the planet in years(y).
Q ≡ The quotient (T^2/a^3) in units of 10^{-34} y^2 m^{-3}.)

Planet	a	T	Q
Mercury	5.79	0.24	2.95
Venus	10.8	0.615	3.00
Earth	15.0	1	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84	2.98
Neptune	450	165	2.99

The law of areas can be understood as a consequence of conservation of angular momentum whch is valid for any central force . A central force is such that the force on the planet is along the vector joining the Sun and the planet. Let the Sun be at the origin and let the position and momentum of the planet be denoted by **r** and **p** respectively. Then the area swept out by the planet of mass m in time interval Δt is (Fig. 7.2) ΔA given by

$$\Delta \mathbf{A} = \tfrac{1}{2}\ (\mathbf{r} \times \mathbf{v}\Delta t) \tag{7.1}$$

Hence

$$\Delta \mathbf{A}/\Delta t = \tfrac{1}{2}\,(\mathbf{r} \times \mathbf{p})/m, \text{ (since } \mathbf{v} = \mathbf{p}/m)$$
$$= \mathbf{L}/(2\,m) \tag{7.2}$$

where **v** is the velocity, **L** is the angular momentum equal to (**r** × **p**). For a central force, which is directed along **r**, **L** is a constant as the planet goes around. Hence, $\Delta \mathbf{A}/\Delta t$ is a constant according to the last equation. This is the law of areas. Gravitation is a central force and hence the law of areas follows.

▶ ***Example 7.1*** Let the speed of the planet at the perihelion *P* in Fig. 7.1(a) be v_P and the Sun-planet distance SP be r_P. Relate {r_P, v_P} to the corresponding quantities at the aphelion {r_A, v_A}. Will the planet take equal times to traverse *BAC* and *CPB* ?

Answer The magnitude of the angular momentum at *P* is $L_p = m_p r_p v_p$, since inspection tells us that $\mathbf{r}_p$ and $\mathbf{v}_p$ are mutually perpendicular. Similarly, $L_A = m_p\, r_A\, v_A$. From angular momentum conservation

$$m_p r_p v_p = m_p r_A v_A$$

or

$$\frac{v_p}{v_A} = \frac{r_A}{r_p}$$

Since $r_A > r_p$, $v_p > v_A$.

The area *SBAC* bounded by the ellipse and the radius vectors *SB* and *SC* is larger than SBPC in Fig. 7.1. From Kepler's second law, equal areas are swept in equal times. Hence the planet will take a longer time to traverse *BAC* than *CPB*. ◀

7.3 UNIVERSAL LAW OF GRAVITATION

Legend has it that observing an apple falling from a tree, Newton was inspired to arrive at an universal law of gravitation that led to an explanation of terrestrial gravitation as well as of Kepler's laws. Newton's reasoning was that the moon revolving in an orbit of radius R_m was subject to a centripetal acceleration due to earth's gravity of magnitude

$$a_m = \frac{V^2}{R_m} = \frac{4\pi^2 R_m}{T^2} \tag{7.3}$$

where *V* is the speed of the moon related to the time period *T* by the relation $V = 2\pi R_m / T$. The time period *T* is about 27.3 days and R_m was already known then to be about 3.84 10^8m. If we substitute these numbers in Eq. (7.3), we get a value of a_m much smaller than the value of acceleration due to gravity g on the surface of the earth, arising also due to earth's gravitational attraction.

This clearly shows that the force due to earth's gravity decreases with distance. If one assumes that the gravitational force due to the earth decreases in proportion to the inverse square of the distance from the centre of the earth, we will have $a_m \propto R_m^{-2}$; $g \propto R_E^{-2}$ and we get

$$\frac{g}{a_m} = \frac{R_m^2}{R_E^2} \simeq 3600 \tag{7.4}$$

in agreement with a value of $g \simeq 9.8$ m s^{-2} and the value of a_m from Eq. (7.3). These observations led Newton to propose the following Universal Law of Gravitation :

Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

The quotation is essentially from Newton's famous treatise called 'Mathematical Principles of Natural Philosophy' (Principia for short).

Stated Mathematically, Newton's gravitation law reads : The force **F** on a point mass m_2 due to another point mass m_1 has the magnitude

$$|\mathbf{F}| = G\,\frac{m_1\, m_2}{r^2} \tag{7.5}$$

Equation (7.5) can be expressed in vector form as

$$\mathbf{F} = G\,\frac{m_1\, m_2}{r^2}\left(-\hat{\mathbf{r}}\right) = -G\,\frac{m_1\, m_2}{r^2}\,\hat{\mathbf{r}}$$

$$= -G\,\frac{m_1\, m_2}{|\mathbf{r}|^3}\,\hat{\mathbf{r}}$$

where G is the universal gravitational constant, $\hat{\mathbf{r}}$ is the unit vector from m_1 to m_2 and $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$ as shown in Fig. 7.3.

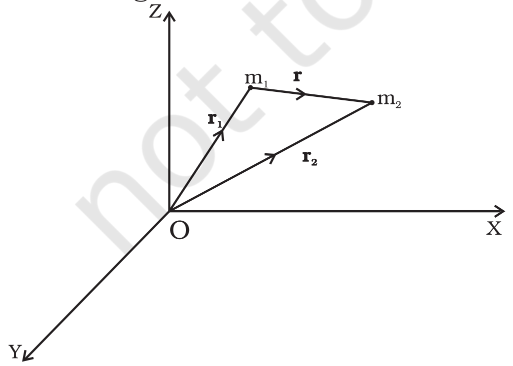


Fig. 7.3 *Gravitational force on m_1 due to m_2 is along* $\mathbf{r}$ *where the vector* $\mathbf{r}$ *is* $(\mathbf{r}_2 - \mathbf{r}_1)$.

The gravitational force is attractive, i.e., the force **F** is along – **r**. The force on point mass m_1 due to m_2 is of course – **F** by Newton's third law. Thus, the gravitational force $\mathbf{F}_{12}$ on the body 1 due to 2 and $\mathbf{F}_{21}$ on the body 2 due to 1 are related as $\mathbf{F}_{12} = -\mathbf{F}_{21}$.

Before we can apply Eq. (7.5) to objects under consideration, we have to be careful since the law refers to **point** masses whereas we deal with extended objects which have finite size. If we have a collection of point masses, the force on any one of them is the vector sum of the gravitational forces exerted by the other point masses as shown in Fig 7.4.

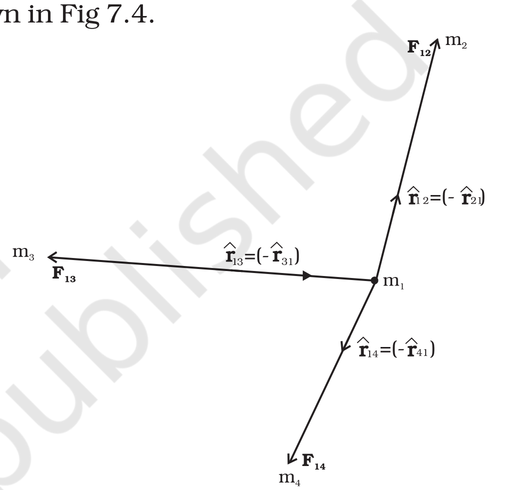


Fig. 7.4 *Gravitational force on point mass m_1 is the vector sum of the gravitational forces exerted by m_2, m_3 and m_4.*

The total force on m_1 is

$$\mathbf{F}_1 = \frac{Gm_2\, m_1}{r_{21}^2}\,\hat{\mathbf{r}}_{21} + \frac{Gm_3\, m_1}{r_{31}^2}\,\hat{\mathbf{r}}_{31} + \frac{Gm_4\, m_1}{r_{41}^2}\,\hat{\mathbf{r}}_{41}$$

▶ ***Example 7.2*** Three equal masses of m kg each are fixed at the vertices of an equilateral triangle ABC.
(a) What is the force acting on a mass $2m$ placed at the centroid G of the triangle?
(b) What is the force if the mass at the vertex A is doubled ?
Take AG = BG = CG = 1 m (see Fig. 7.5)

Answer (a) The angle between GC and the positive x-axis is 30° and so is the angle between GB and the negative x-axis. The individual forces in vector notation are

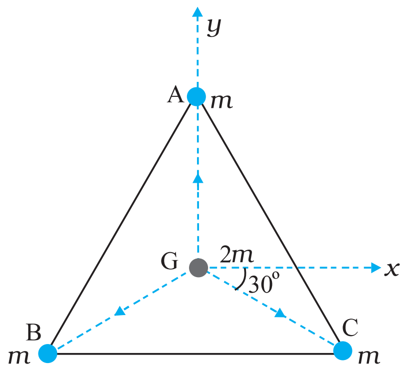


Fig. 7.5 Three equal masses are placed at the three vertices of the Δ ABC. A mass 2m is placed at the centroid G.

$$\mathbf{F}_{GA} = \frac{Gm(2m)}{1}\hat{\mathbf{j}}$$

$$\mathbf{F}_{GB} = \frac{Gm(2m)}{1}\left(-\hat{\mathbf{i}}\cos 30^o - \hat{\mathbf{j}}\sin 30^o\right)$$

$$\mathbf{F}_{GC} = \frac{Gm(2m)}{1}\left(+\hat{\mathbf{i}}\cos 30^o - \hat{\mathbf{j}}\sin 30^o\right)$$

From the principle of superposition and the law of vector addition, the resultant gravitational force $\mathbf{F}_R$ on ($2m$) is

$$\mathbf{F}_R = \mathbf{F}_{GA} + \mathbf{F}_{GB} + \mathbf{F}_{GC}$$

$$\mathbf{F}_R = 2Gm^2\,\hat{\mathbf{j}} + 2Gm^2\left(-\hat{\mathbf{i}}\cos 30^o - \hat{\mathbf{j}}\sin 30^o\right) + 2Gm^2\left(\hat{\mathbf{i}}\cos 30^o - \hat{\mathbf{j}}\sin 30^o\right) = 0$$

Alternatively, one expects on the basis of symmetry that the resultant force ought to be zero.

(b) Now if the mass at vertex A is doubled then

$$F'_{GA} = \frac{G2m.2m}{1}\hat{j} = 4Gm^2\hat{j}$$

$$F'_{GB} = F_{GB} \text{ and } F'_{GC} = F_{GC}$$

$$F'_R = F'_{GA} + F'_{GB} + F'_{GC}$$

$$F'_R = 2\,Gm^2\,\hat{j}$$

For the gravitational force between an extended object (like the earth) and a point mass, Eq. (7.5) is not directly applicable. Each point mass in the extended object will exert a force on the given point mass and these force will not all be in the same direction. We have to add up these forces vectorially for all the point masses in the extended object to get the total force. This is easily done using calculus. For two special cases, a simple law results when you do that :

(1) **The force of attraction between a hollow spherical shell of uniform density and a point mass situated outside is just as if the entire mass of the shell is concentrated at the centre of the shell.**
Qualitatively this can be understood as follows: Gravitational forces caused by the various regions of the shell have components along the line joining the point mass to the centre as well as along a direction prependicular to this line. The components prependicular to this line cancel out when summing over all regions of the shell leaving only a resultant force along the line joining the point to the centre. The magnitude of this force works out to be as stated above.

(2) **The force of attraction due to a hollow spherical shell of uniform density, on a point mass situated inside it is zero.**
Qualitatively, we can again understand this result. Various regions of the spherical shell attract the point mass inside it in various directions. These forces cancel each other completely.

7.4 THE GRAVITATIONAL CONSTANT

The value of the gravitational constant G entering the Universal law of gravitation can be determined experimentally and this was first done by English scientist Henry Cavendish in 1798. The apparatus used by him is schematically shown in Fig.7.6

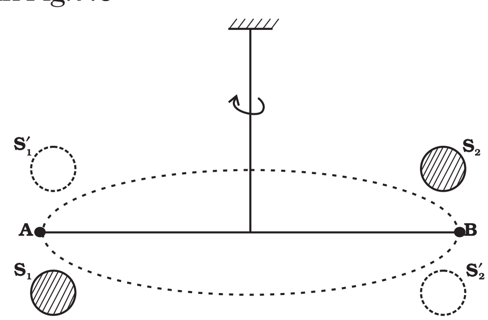


Fig. 7.6 Schematic drawing of Cavendish's experiment. S_1 and S_2 are large spheres which are kept on either side (shown shades) of the masses at A and B. When the big spheres are taken to the other side of the masses (shown by dotted circles), the bar AB rotates a little since the torque reverses direction. The angle of rotation can be measured experimentally.

The bar AB has two small lead spheres attached at its ends. The bar is suspended from a rigid support by a fine wire. Two large lead spheres are brought close to the small ones but on opposite sides as shown. The big spheres attract the nearby small ones by equal and opposite force as shown. There is no net force on the bar but only a torque which is clearly equal to F times the length of the bar,where F is the force of attraction between a big sphere and its neighbouring small sphere. Due to this torque, the suspended wire gets twisted till such time as the restoring torque of the wire equals the gravitational torque . If θ is the angle of twist of the suspended wire, the restoring torque is proportional to θ, equal to $\tau\theta$. Where τ is the restoring couple per unit angle of twist. τ can be measured independently e.g. by applying a known torque and measuring the angle of twist. The gravitational force between the spherical balls is the same as if their masses are concentrated at their centres. Thus if d is the separation between the centres of the big and its neighbouring small ball, M and m their masses, the gravitational force between the big sphere and its neighouring small ball is.

$$F = G\frac{Mm}{d^2} \tag{7.6}$$

If L is the length of the bar AB , then the torque arising out of F is F multiplied by L. At equilibrium, this is equal to the restoring torque and hence

$$G\frac{Mm}{d^2}L = \tau\,\theta \tag{7.7}$$

Observation of θ thus enables one to calculate G from this equation.

Since Cavendish's experiment, the measurement of G has been refined and the currently accepted value is

$$G = 6.67\times10^{-11}\ \mathrm{N\,m^2/kg^2} \tag{7.8}$$

7.5 ACCELERATION DUE TO GRAVITY OF THE EARTH

The earth can be imagined to be a sphere made of a large number of concentric spherical shells with the smallest one at the centre and the largest one at its surface. A point outside the earth is obviously outside all the shells. Thus, all the shells exert a gravitational force at the point outside just as if their masses are concentrated at their common centre according to the result stated in section 7.3. The total mass of all the shells combined is just the mass of the earth. Hence, at a point outside the earth, the gravitational force is just as if its entire mass of the earth is concentrated at its centre.

For a point inside the earth, the situation is different. This is illustrated in Fig. 7.7.

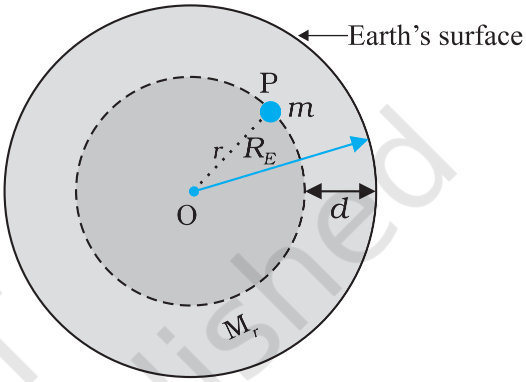


Fig. 7.7 *The mass m is in a mine located at a depth d below the surface of the Earth of mass M_E and radius R_E. We treat the Earth to be spherically symmetric.*

Again consider the earth to be made up of concentric shells as before and a point mass m situated at a distance r from the centre. The point P lies outside the sphere of radius r. For the shells of radius greater than r, the point P lies inside. Hence according to result stated in the last section, they exert no gravitational force on mass m kept at P. The shells with radius $\leq r$ make up a sphere of radius r for which the point P lies on the surface. This smaller sphere therefore exerts a force on a mass m at P as if its mass M_r is concentrated at the centre. Thus the force on the mass m at P has a magnitude

$$F = \frac{Gm\,(M_r)}{r^2} \tag{7.9}$$

We assume that the entire earth is of uniform density and hence its mass is $M_E = \frac{4\pi}{3}R_E^3\,\rho$ where M_E is the mass of the earth R_E is its radius and ρ is the density. On the other hand the mass of the sphere M_r of radius r is $\frac{4\pi}{3}\rho r^3$ and hence

$$F = G\,m\left(\frac{4p}{3}\,r\right)\frac{r^3}{r^2} = G\,m\left(\frac{M_E}{R_E^3}\right)\frac{r^3}{r^2}$$

$$= \frac{G\,m\,M_E}{{R_E}^3}\,r \qquad (7.10)$$

If the mass m is situated on the surface of earth, then $r = R_E$ and the gravitational force on it is, from Eq. (7.10)

$$F = G\,\frac{M_E m}{R_E^2} \qquad (7.11)$$

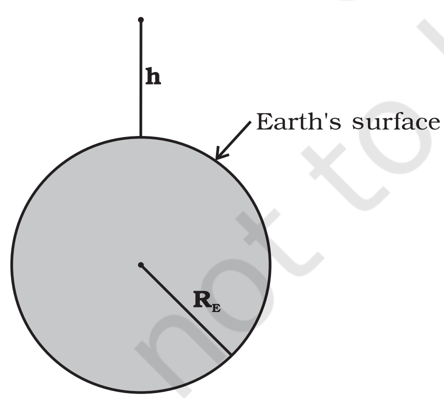
The acceleration experienced by the mass m, which is usually denoted by the symbol g is related to F by Newton's 2[nd] law by relation $F = mg$. Thus

$$g = \frac{F}{m} = \frac{GM_E}{R_E^2} \qquad (7.12)$$

Acceleration g is readily measurable. R_E is a known quantity. The measurement of G by Cavendish's experiment (or otherwise), combined with knowledge of g and R_E enables one to estimate M_E from Eq. (7.12). This is the reason why there is a popular statement regarding Cavendish : "Cavendish weighed the earth".

7.6 ACCELERATION DUE TO GRAVITY BELOW AND ABOVE THE SURFACE OF EARTH

Consider a point mass m at a height h above the surface of the earth as shown in Fig. 7.8(a). The radius of the earth is denoted by R_E. Since this point is outside the earth,



(a)

Fig. 7.8 (a) *g at a height h above the surface of the earth.*

its distance from the centre of the earth is $(R_E + h)$. If $F(h)$ denoted the magnitude of the force on the point mass m, we get from Eq. (7.5) :

$$F(h) = \frac{GM_E m}{(R_E + h)^2} \qquad (7.13)$$

The acceleration experienced by the point mass is $F(h)/m \equiv g(h)$ and we get

$$g(h) = \frac{F(h)}{m} = \frac{GM_E}{(R_E + h)^2}\,. \qquad (7.14)$$

This is clearly less than the value of g on the surface of earth : $g = \frac{GM_E}{R_E^2}$. For $h << R_E$, we can expand the RHS of Eq. (7.14) :

$$g(h) = \frac{GM_E}{R_E^2(1 + h/R_E)^2} = g\,(1 + h/R_E)^{-2}$$

For $\frac{h}{R_E} << 1$, using binomial expression,

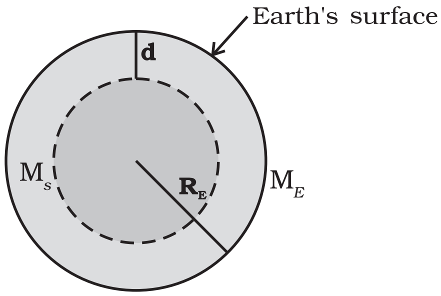
$$g\,(h) \cong g\left(1 - \frac{2h}{R_E}\right). \qquad (7.15)$$

Equation (7.15) thus tells us that for small heights h above the value of g decreases by a factor $(1 - 2h/R_E)$.

Now, consider a point mass m at a depth d below the surface of the earth (Fig. 7.8(b)), so that its distance from the centre of the earth is $(R_E - d)$ as shown in the figure. The earth can be thought of as being composed of a smaller sphere of radius $(R_E - d)$ and a spherical shell of thickness d. The force on m due to the outer shell of thickness d is zero because the result quoted in the previous section. As far as the smaller sphere of radius $(R_E - d)$ is concerned, the point mass is outside it and hence according to the result quoted earlier, the force due to this smaller sphere is just as if the entire mass of the smaller sphere is concentrated at the centre. If M_s is the mass of the smaller sphere, then,

$$M_s/M_E = (R_E - d)^3\ /\ R_E^{\,3} \qquad (7.16)$$

Since mass of a sphere is proportional to be cube of its radius.



(b)

Fig. 7.8 (b) *g at a depth d. In this case only the smaller sphere of radius (R_E–d) contributes to g.*

Thus the force on the point mass is

$$F(d) = G\ M_s\ m\ /\ (R_E - d)^2 \quad (7.17)$$

Substituting for M_s from above, we get

$$F(d) = G\ M_E\ m\,(R_E - d)\ /\ R_E^3 \quad (7.18)$$

and hence the acceleration due to gravity at a depth d,

$$g(d) = \frac{F(d)}{m} \text{ is}$$

$$g(d) = \frac{F(d)}{m} = \frac{GM_E}{R_E^3}\ (R_E - d)$$

$$= g\frac{R_E - d}{R_E} = g(1 - d/R_E) \quad (7.19)$$

Thus, as we go down below earth's surface, the acceleration due gravity decreases by a factor $(1 - d/R_E)$. The remarkable thing about acceleration due to earth's gravity is that it is maximum on its surface decreasing whether you go up or down.

7.7 GRAVITATIONAL POTENTIAL ENERGY

We had discussed earlier the notion of potential energy as being the energy stored in the body at its given position. If the position of the particle changes on account of forces acting on it, then the change in its potential energy is just the amount of work done on the body by the force. As we had discussed earlier, forces for which the work done is independent of the path are the conservative forces.

The force of gravity is a conservative force and we can calculate the potential energy of a body arising out of this force, called the gravitational potential energy. Consider points close to the surface of earth, at distances from the surface much smaller than the radius of the earth. In such cases, the force of gravity is practically a constant equal to mg, directed towards the centre of the earth. If we consider a point at a height h_1 from the surface of the earth and another point vertically above it at a height h_2 from the surface, the work done in lifting the particle of mass m from the first to the second position is denoted by W_{12}

$$W_{12} = \text{Force} \times \text{displacement}$$
$$= mg\ (h_2 - h_1) \quad (7.20)$$

If we associate a potential energy $W(h)$ at a point at a height h above the surface such that

$$W(h) = mgh + W_o \quad (7.21)$$

(where W_o = constant) ;
then it is clear that

$$W_{12} = W(h_2) - W(h_1) \quad (7.22)$$

The work done in moving the particle is just the difference of potential energy between its final and initial positions.Observe that the constant W_o cancels out in Eq. (7.22). Setting $h = 0$ in the last equation, we get $W(h = 0) = W_o$. $h = 0$ means points on the surface of the earth. Thus, W_o is the potential energy on the surface of the earth.

If we consider points at arbitrary distance from the surface of the earth, the result just derived is not valid since the assumption that the gravitational force mg is a constant is no longer valid. However, from our discussion we know that a point outside the earth, the force of gravitation on a particle directed towards the centre of the earth is

$$F = \frac{G M_E m}{r^2} \quad (7.23)$$

where M_E = mass of earth, m = mass of the particle and r its distance from the centre of the earth. If we now calculate the work done in lifting a particle from $r = r_1$ to $r = r_2$ ($r_2 > r_1$) along a vertical path, we get instead of Eq. (7.20)

$$W_{12} = \int_{r_1}^{r_2} \frac{G M m}{r^2} \mathrm{d}r$$

$$= -G M_E m \left(\frac{1}{r_2} - \frac{1}{r_1}\right) \quad (7.24)$$

In place of Eq. (7.21), we can thus associate a potential energy $W(r)$ at a distance r, such that

$$W(r) = -\frac{G M_E m}{r} + W_1 , \qquad (7.25)$$

valid for $r > R$,

so that once again $W_{12} = W(r_2) - W(r_1)$. Setting r = infinity in the last equation, we get W (r = infinity) = W_1 . Thus, W_1 is the potential energy at infinity. One should note that only the difference of potential energy between two points has a definite meaning from Eqs. (7.22) and (7.24). One conventionally sets W_1 equal to zero, so that the potential energy at a point is just the amount of work done in displacing the particle from infinity to that point.

We have calculated the potential energy at a point of a particle due to gravitational forces on it due to the earth and it is proportional to the mass of the particle. The gravitational potential due to the gravitational force of the earth is defined as the potential energy of a particle of unit mass at that point. From the earlier discussion, we learn that the gravitational potential energy associated with two particles of masses m_1 and m_2 separated by distance by a distance r is given by

$$V = -\frac{G m_1 m_2}{r} \quad \text{(if we choose } V = 0 \text{ as } r \rightarrow \infty\text{)}$$

It should be noted that an isolated system of particles will have the total potential energy that equals the sum of energies (given by the above equation) for all possible pairs of its constituent particles. This is an example of the application of the superposition principle.

Example 7.3 Find the potential energy of a system of four particles placed at the vertices of a square of side l. Also obtain the potential at the centre of the square.

Answer Consider four masses each of mass m at the corners of a square of side l; See Fig. 7.9. We have four mass pairs at distance l and two diagonal pairs at distance $\sqrt{2}\, l$

Hence,

$$W(r) = -4\frac{G m^2}{l} - 2\frac{G m^2}{\sqrt{2}\, l}$$

$$= -\frac{2 G m^2}{l}\left(2 + \frac{1}{\sqrt{2}}\right) = -5.41 \frac{G m^2}{l}$$

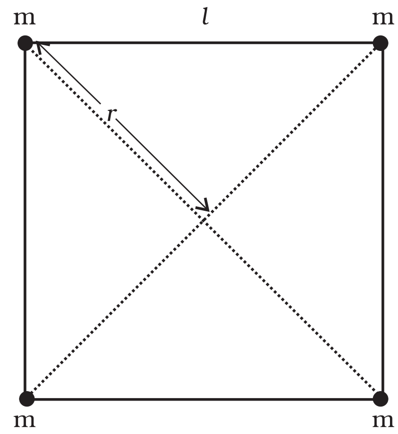


Fig. 7.9

The gravitational potential at the centre of the square $\left(r = \sqrt{2}\, l/2\right)$ is

$$U(r) = -4\sqrt{2}\,\frac{G m}{l}.$$

7.8 ESCAPE SPEED

If a stone is thrown by hand, we see it falls back to the earth. Of course using machines we can shoot an object with much greater speeds and with greater and greater initial speed, the object scales higher and higher heights. A natural query that arises in our mind is the following: 'can we throw an object with such high initial speeds that it does not fall back to the earth?'

The principle of conservation of energy helps us to answer this question. Suppose the object did reach infinity and that its speed there was V_f. The energy of an object is the sum of potential and kinetic energy. As before W_1 denotes that gravitational potential energy of the object at infinity. The total energy of the projectile at infinity then is

$$E\,(\infty) = W_1 + \frac{m V_f^2}{2} \qquad (7.26)$$

If the object was thrown initially with a speed V_i from a point at a distance $(h+R_E)$ from the centre of the earth (R_E = radius of the earth), its energy initially was

$$E\,(h + R_E) = \frac{1}{2} m V_i^2 - \frac{G m M_E}{(h + R_E)} + W_1 \qquad (7.27)$$

By the principle of energy conservation Eqs. (7.26) and (7.27) must be equal. Hence

$$\frac{mV_i^2}{2} - \frac{GmM_E}{(h+R_E)} = \frac{mV_f^2}{2} \qquad (7.28)$$

The R.H.S. is a positive quantity with a minimum value zero hence so must be the L.H.S. Thus, an object can reach infinity as long as V_i is such that

$$\frac{mV_i^2}{2} - \frac{GmM_E}{(h+R_E)} \geq 0 \qquad (7.29)$$

The minimum value of V_i corresponds to the case when the L.H.S. of Eq. (7.29) equals zero. Thus, the minimum speed required for an object to reach infinity (i.e. escape from the earth) corresponds to

$$\frac{1}{2}m\left(V_i^2\right)_{\min} = \frac{GmM_E}{h+R_E} \qquad (7.30)$$

If the object is thrown from the surface of the earth, $h = 0$, and we get

$$(V_i)_{\min} = \sqrt{\frac{2GM_E}{R_E}} \qquad (7.31)$$

Using the relation $g = GM_E / R_E^2$, we get

$$(V_i)_{\min} = \sqrt{2gR_E} \qquad (7.32)$$

Using the value of g and R_E, numerically $(V_i)_{\min} \approx 11.2$ km/s. This is called the escape speed, sometimes loosely called the escape velocity.

Equation (7.32) applies equally well to an object thrown from the surface of the moon with g replaced by the acceleration due to Moon's gravity on its surface and r_E replaced by the radius of the moon. Both are smaller than their values on earth and the escape speed for the moon turns out to be 2.3 km/s, about five times smaller. This is the reason that moon has no atmosphere. Gas molecules if formed on the surface of the moon having velocities larger than this will escape the gravitational pull of the moon.

Example 7.4 Two uniform solid spheres of equal radii R, but mass M and 4 M have a centre to centre separation 6 R, as shown in Fig. 7.10. The two spheres are held fixed. A projectile of mass m is projected from the surface of the sphere of mass M directly towards the centre of the second sphere. Obtain an expression for the minimum speed v of the projectile so that it reaches the surface of the second sphere.

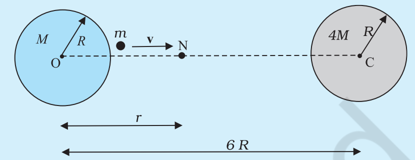


Fig. 7.10

Answer The projectile is acted upon by two mutually opposing gravitational forces of the two spheres. The neutral point N (see Fig. 7.10) is defined as the position where the two forces cancel each other exactly. If ON = r, we have

$$\frac{G\,M\,m}{r^2} = \frac{4\,G\,M\,m}{(6R-r)^2}$$

$(6R - r)^2 = 4r^2$

$6R - r = \pm 2r$

$r = 2R$ or $- 6R$.

The neutral point $r = -6R$ does not concern us in this example. Thus ON = r = $2R$. It is sufficient to project the particle with a speed which would enable it to reach N. Thereafter, the greater gravitational pull of $4M$ would suffice. The mechanical energy at the surface of M is

$$E_i = \frac{1}{2}m\,v^2 - \frac{G\,M\,m}{R} - \frac{4\,G\,M\,m}{5\,R}.$$

At the neutral point N, the speed approaches zero. The mechanical energy at N is purely potential.

$$E_N = -\frac{G\,M\,m}{2\,R} - \frac{4\,G\,M\,m}{4\,R}.$$

From the principle of conservation of mechanical energy

$$\frac{1}{2}v^2 - \frac{GM}{R} - \frac{4GM}{5R} = -\frac{GM}{2R} - \frac{GM}{R}$$

or

$$v^2 = \frac{2GM}{R}\left(\frac{4}{5}-\frac{1}{2}\right)$$

$$v = \left(\frac{3GM}{5R}\right)^{1/2}$$

A point to note is that the speed of the projectile is zero at N, but is nonzero when it strikes the heavier sphere 4 M. The calculation of this speed is left as an exercise to the students.

7.9 EARTH SATELLITES

Earth satellites are objects which revolve around the earth. Their motion is very similar to the motion of planets around the Sun and hence Kepler's laws of planetary motion are equally applicable to them. In particular, their orbits around the earth are circular or elliptic. Moon is the only natural satellite of the earth with a near circular orbit with a time period of approximately 27.3 days which is also roughly equal to the rotational period of the moon about its own axis. Since, 1957, advances in technology have enabled many countries including India to launch artificial earth satellites for practical use in fields like telecommunication, geophysics and meteorology.

We will consider a satellite in a circular orbit of a distance $(R_E + h)$ from the centre of the earth, where R_E = radius of the earth. If m is the mass of the satellite and V its speed, the centripetal force required for this orbit is

$$\text{F(centripetal)} = \frac{mV^2}{(R_E + h)} \qquad (7.33)$$

directed towards the centre. This centripetal force is provided by the gravitational force, which is

$$\text{F(gravitation)} = \frac{G\,m\,M_E}{(R_E + h)^2} \qquad (7.34)$$

where M_E is the mass of the earth.

Equating R.H.S of Eqs. (7.33) and (7.34) and cancelling out m, we get

$$V^2 = \frac{G\,M_E}{(R_E + h)} \qquad (7.35)$$

Thus V decreases as h increases. From equation (7.35),the speed V for $h = 0$ is

$$V^2 \;(h = 0) = GM/R_E = gR_E \qquad (7.36)$$

where we have used the relation $g = GM/R_E^2$. In every orbit, the satellite traverses a distance $2\pi(R_E + h)$ with speed V. Its time period T therefore is

$$T = \frac{2\pi(R_E + h)}{V} = \frac{2\pi(R_E + h)^{3/2}}{\sqrt{G\,M_E}} \qquad (7.37)$$

on substitution of value of V from Eq. (7.35). Squaring both sides of Eq. (7.37), we get

$$T^2 = k\;(R_E + h)^3 \;\; (\text{where } k = 4\pi^2/GM_E) \qquad (7.38)$$

which is Kepler's law of periods, as applied to motion of satellites around the earth. For a satellite very close to the surface of earth h can be neglected in comparison to R_E in Eq. (7.38). Hence, for such satellites, T is T_o, where

$$T_0 = 2\pi\sqrt{R_E/g} \qquad (7.39)$$

If we substitute the numerical values $g \simeq 9.8$ m s^{-2} and R_E = 6400 km., we get

$$T_0 = 2\pi\sqrt{\frac{6.4\times10^6}{9.8}}\ \text{s}$$

Which is approximately 85 minutes.

> **Example 7.5** The planet Mars has two moons, phobos and delmos. (i) phobos has a period 7 hours, 39 minutes and an orbital radius of 9.4×10^3 km. Calculate the mass of mars. (ii) Assume that earth and mars move in circular orbits around the sun, with the martian orbit being 1.52 times the orbital radius of the earth. What is the length of the martian year in days ?

Answer (i) We employ Eq. (7.38) with the sun's mass replaced by the martian mass M_m

$$T^2 = \frac{4\pi^2}{GM_m}R^3$$

$$M_m = \frac{4\pi^2}{G}\frac{R^3}{T^2}$$

$$= \frac{4\times(3.14)^2\times(9.4)^3\times10^{18}}{6.67\times10^{-11}\times(459\times60)^2}$$

$$M_m = \frac{4\times(3.14)^2\times(9.4)^3\times10^{18}}{6.67\times(4.59\times6)^2\times10^{-5}}$$

$= 6.48 \times 10^{23}$ kg.

(ii) Once again Kepler's third law comes to our aid,

$$\frac{T_M^2}{T_E^2} = \frac{R_{MS}^3}{R_{ES}^3}$$

where R_{MS} is the mars -sun distance and R_{ES} is the earth-sun distance.

$\therefore\ T_M = (1.52)^{3/2} \times 365$

$= 684$ days

We note that the orbits of all planets except Mercury and Mars are very close to being circular. For example, the ratio of the semi-minor to semi-major axis for our Earth is, $b/a = 0.99986$.

Example 7.6 Weighing the Earth : You are given the following data: $g = 9.81\ \text{ms}^{-2}$, $R_E = 6.37\times10^6$ m, the distance to the moon $R = 3.84\times10^8$ m and the time period of the moon's revolution is 27.3 days. Obtain the mass of the Earth M_E in two different ways.

Answer From Eq. (7.12) we have

$$M_E = \frac{g R_E^2}{G}$$

$$= \frac{9.81\times\left(6.37\times10^6\right)^2}{6.67\times10^{-11}}$$

$= 5.97\times 10^{24}$ kg.

The moon is a satellite of the Earth. From the derivation of Kepler's third law [see Eq. (7.38)]

$$T^2 = \frac{4\pi^2R^3}{G M_E}$$

$$M_E = \frac{4\pi^2R^3}{G T^2}$$

$$= \frac{4\times3.14\times3.14\times(3.84)^3\times10^{24}}{6.67\times10^{-11}\times(27.3\times24\times60\times60)^2}$$

$$= 6.02\times10^{24}\ \text{kg}$$

Both methods yield almost the same answer, the difference between them being less than 1%.

Example 7.7 Express the constant k of Eq. (7.38) in days and kilometres. Given $k = 10^{-13}\ \text{s}^2\ \text{m}^{-3}$. The moon is at a distance of 3.84×10^5 km from the earth. Obtain its time-period of revolution in days.

Answer Given

$k = 10^{-13}\ \text{s}^2\ \text{m}^{-3}$

$$= 10^{-13}\left[\frac{1}{(24\times60\times60)^2}\text{d}^2\right]\left[\frac{1}{(1/1000)^3\,\text{km}^3}\right]$$

$= 1.33\times10^{-14}\ \text{d}^2\ \text{km}^{-3}$

Using Eq. (7.38) and the given value of k, the time period of the moon is

$T^2 = (1.33\times10^{-14})(3.84\times10^5)^3$

$T = 27.3$ d

Note that Eq. (7.38) also holds for elliptical orbits if we replace (R_E+h) by the semi-major axis of the ellipse. The earth will then be at one of the foci of this ellipse.

7.10 ENERGY OF AN ORBITING SATELLITE

Using Eq. (7.35), the kinetic energy of the satellite in a circular orbit with speed v is

$$K{\cdot}E = \frac{1}{2}m\,v^2$$

$$= \frac{Gm\,M_E}{2(R_E+h)}, \tag{7.40}$$

Considering gravitational potential energy at infinity to be zero, the potential energy at distance (R_e+h) from the centre of the earth is

$$P.E = -\frac{G\,m\,M_E}{(R_E+h)} \tag{7.41}$$

The K.E is positive whereas the P.E is negative. However, in magnitude the K.E. is half the P.E, so that the total E is

$$E = K.E + P.E = -\frac{G\,m\,M_E}{2(R_E+h)} \tag{7.42}$$

The total energy of an circularly orbiting satellite is thus negative, with the potential energy being negative but twice is magnitude of the positive kinetic energy.

When the orbit of a satellite becomes elliptic, both the *K.E.* and *P.E.* vary from point to point. The total energy which remains constant is negative as in the circular orbit case. This is what we expect, since as we have discussed before if the total energy is positive or zero, the object escapes to infinity. Satellites are always at finite distance from the earth and hence their energies cannot be positive or zero.

Example 7.8 A 400 kg satellite is in a circular orbit of radius $2R_E$ about the Earth. How much energy is required to transfer it to a circular orbit of radius $4R_E$? What are the changes in the kinetic and potential energies ?

Answer Initially,

$$E_i = -\frac{G\,M_E\,m}{4\,R_E}$$

While finally

$$E_f = -\frac{G\,M_E\,m}{8\,R_E}$$

The change in the total energy is

$$\Delta E = E_f - E_i$$

$$= \frac{G\,M_E\,m}{8\,R_E} = \left(\frac{G\,M_E}{R_E^2}\right)\frac{m\,R_E}{8}$$

$$\Delta E = \frac{g\,m\,R_E}{8} = \frac{9.81 \times 400 \times 6.37 \times 10^6}{8} = 3.13 \times 10^9\,\text{J}$$

The kinetic energy is reduced and it mimics ΔE, namely, $\Delta K = K_f - K_i = -3.13 \times 10^9$ J.

The change in potential energy is twice the change in the total energy, namely

$$\Delta V = V_f - V_i = -6.25 \times 10^9\,\text{J}$$

SUMMARY

1. Newton's law of universal gravitation states that the gravitational force of attraction between any two particles of masses m_1 and m_2 separated by a distance r has the magnitude

$$F = G\frac{m_1 m_2}{r^2}$$

where G is the universal gravitational constant, which has the value 6.672×10^{-11} N m^2 kg^{-2}.

2. If we have to find the resultant gravitational force acting on the particle m due to a number of masses $M_1, M_2, \ldots M_n$ etc. we use the principle of superposition. Let $F_1, F_2, \ldots F_n$ be the individual forces due to $M_1, M_2, \ldots M_n$, each given by the law of gravitation. From the principle of superposition each force acts independently and uninfluenced by the other bodies. The resultant force F_R is then found by vector addition

$$F_R = F_1 + F_2 + \ldots\ldots + F_n = \sum_{i=1}^{n} F_i$$

where the symbol 'Σ' stands for summation.

3. Kepler's laws of planetary motion state that
   (a) All planets move in elliptical orbits with the Sun at one of the focal points
   (b) The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals. This follows from the fact that the force of gravitation on the planet is central and hence angular momentum is conserved.
   (c) The square of the orbital period of a planet is proportional to the cube of the semi-major axis of the elliptical orbit of the planet

   The period T and radius R of the circular orbit of a planet about the Sun are related by

$$T^2 = \left(\frac{4\pi^2}{G\,M_s}\right)R^3$$

where M_s is the mass of the Sun. Most planets have nearly circular orbits about the Sun. For elliptical orbits, the above equation is valid if R is replaced by the semi-major axis, a.

4. The acceleration due to gravity.
   (a) at a height h above the earth's surface

$$g(h) = \frac{G\,M_E}{(R_E + h)^2}$$

$$\approx \frac{G\,M_E}{R_E^2}\left(1 - \frac{2h}{R_E}\right) \quad \text{for } h << R_E$$

$$g(h) = g(0)\left(1 - \frac{2h}{R_E}\right) \quad \text{where } g(0) = \frac{G\,M_E}{R_E^2}$$

(b) at depth d below the earth's surface is

$$g(d) = \frac{G\,M_E}{R_E^2}\left(1 - \frac{d}{R_E}\right) = g(0)\left(1 - \frac{d}{R_E}\right)$$

5. The gravitational force is a conservative force, and therefore a potential energy function can be defined. The *gravitational potential energy* associated with two particles separated by a distance r is given by

$$V = -\frac{G\,m_1\,m_2}{r}$$

where V is taken to be zero at $r \to \infty$. The total potential energy for a system of particles is the sum of energies for all pairs of particles, with each pair represented by a term of the form given by above equation. This prescription follows from the principle of superposition.

6. If an isolated system consists of a particle of mass m moving with a speed v in the vicinity of a massive body of mass M, the total mechanical energy of the particle is given by

$$E = \frac{1}{2}m\,v^2 - \frac{G\,M\,m}{r}$$

That is, the total mechanical energy is the sum of the kinetic and potential energies. The total energy is a constant of motion.

7. If m moves in a circular orbit of radius a about M, where $M >> m$, the total energy of the system is

$$E = -\frac{G\,M\,m}{2a}$$

with the choice of the arbitrary constant in the potential energy given in the point 5., above. The total energy is negative for any bound system, that is, one in which the orbit is closed, such as an elliptical orbit. The kinetic and potential energies are

$$K = \frac{G\,M\,m}{2a}$$

$$V = -\frac{G\,M\,m}{a}$$

8. The escape speed from the surface of the earth is

$$v_e = \sqrt{\frac{2\,G\,M_E}{R_E}} = \sqrt{2gR_E}$$

and has a value of 11.2 km s^{-1}.

9. If a particle is outside a uniform spherical shell or solid sphere with a spherically symmetric internal mass distribution, the sphere attracts the particle as though the mass of the sphere or shell were concentrated at the centre of the sphere.

10. If a particle is inside a uniform spherical shell, the gravitational force on the particle is zero. If a particle is inside a homogeneous solid sphere, the force on the particle acts toward the centre of the sphere. This force is exerted by the spherical mass interior to the particle.

Physical Quantity	Symbol	Dimensions	Unit	Remarks
Gravitational Constant	G	$[M^{-1}\,L^3\,T^{-2}]$	N m^2 kg^{-2}	6.67×10^{-11}
Gravitational Potential Energy	$V(r)$	$[M\,L^2\,T^{-2}]$	J	$-\frac{GMm}{r}$ (scalar)
Gravitational Potential	$U(r)$	$[L^2\,T^{-2}]$	J kg^{-1}	$-\frac{GM}{r}$ (scalar)
Gravitational Intensity	$\mathbf{E}$ or $\mathbf{g}$	$[L\,T^{-2}]$	m s^{-2}	$\frac{GM}{r^2}\hat{r}$ (vector)

POINTS TO PONDER

1. In considering motion of an object under the gravitational influence of another object the following quantities are conserved:
   (a) Angular momentum
   (b) Total mechanical energy
   Linear momentum is **not** conserved
2. Angular momentum conservation leads to Kepler's second law. However, it is not special to the inverse square law of gravitation. It holds for any central force.
3. In Kepler's third law (see Eq. (7.1) and $T^2 = K_S R^3$. The constant K_S is the same for all planets in circular orbits. This applies to satellites orbiting the Earth [(Eq. (7.38)].
4. An astronaut experiences weightlessness in a space satellite. This is not because the gravitational force is small at that location in space. It is because both the astronaut and the satellite are in "free fall" towards the Earth.
5. The *gravitational potential energy* associated with two particles separated by a distance r is given by
$$V = -\frac{G m_1 m_2}{r} + \text{constant}$$
   The constant can be given any value. The simplest choice is to take it to be zero. With this choice
$$V = -\frac{G m_1 m_2}{r}$$
   This choice implies that $V \to 0$ as $r \to \infty$. Choosing location of zero of the gravitational energy is the same as choosing the arbitrary constant in the potential energy. Note that the gravitational force is not altered by the choice of this constant.
6. The total mechanical energy of an object is the sum of its kinetic energy (which is always positive) and the potential energy. Relative to infinity (i.e. if we presume that the potential energy of the object at infinity is zero), the gravitational potential energy of an object is negative. The total energy of a satellite is negative.
7. The commonly encountered expression $m g h$ for the potential energy is actually an approximation to the difference in the gravitational potential energy discussed in the point 6, above.
8. Although the gravitational force between two particles is central, the force between two finite rigid bodies is not necessarily along the line joining their centre of mass. For a spherically symmetric body however the force on a particle external to the body is as if the mass is concentrated at the centre and this force is therefore central.
9. The gravitational force on a particle inside a spherical shell is zero. However, (unlike a metallic shell which shields electrical forces) the shell does not shield other bodies outside it from exerting gravitational forces on a particle inside. *Gravitational shielding is not possible.*

EXERCISES

7.1 Answer the following :
(a) You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means ?
(b) An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity ?
(c) If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. (you can check this yourself using the data available in the succeeding exercises). However, the tidal effect of the moon's pull is greater than the tidal effect of sun. Why ?

7.2 Choose the correct alternative :
(a) Acceleration due to gravity increases/decreases with increasing altitude.
(b) Acceleration due to gravity increases/decreases with increasing depth (assume the earth to be a sphere of uniform density).
(c) Acceleration due to gravity is independent of mass of the earth/mass of the body.
(d) The formula $-G\,Mm(1/r_2 - 1/r_1)$ is more/less accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the earth.

7.3 Suppose there existed a planet that went around the Sun twice as fast as the earth. What would be its orbital size as compared to that of the earth ?

7.4 Io, one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. Show that the mass of Jupiter is about one-thousandth that of the sun.

7.5 Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution ? Take the diameter of the Milky Way to be 10^5 ly.

7.6 Choose the correct alternative:
(a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.
(b) The energy required to launch an orbiting satellite out of earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

7.7 Does the escape speed of a body from the earth depend on (a) the mass of the body, (b) the location from where it is projected, (c) the direction of projection, (d) the height of the location from where the body is launched?

7.8 A comet orbits the sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed, (b) angular speed, (c) angular momentum, (d) kinetic energy, (e) potential energy, (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.

7.9 Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem.

7.10 In the following two exercises, choose the correct answer from among the given ones: The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig 7.11) (i) a, (ii) b, (iii) c, (iv) 0.

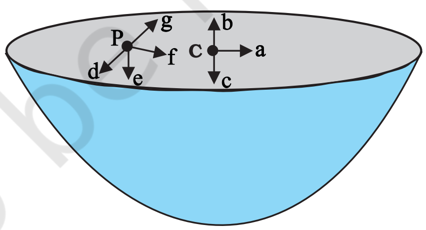


Fig. 7.11

7.11 For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.

7.12 A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero ? Mass of the sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg. Neglect the effect of other planets etc. (orbital radius = 1.5×10^{11} m).

7.13 How will you 'weigh the sun', that is estimate its mass? The mean orbital radius of the earth around the sun is 1.5×10^8 km.

7.14 A saturn year is 29.5 times the earth year. How far is the saturn from the sun if the earth is 1.50×10^8 km away from the sun ?

7.15 A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth ?

7.16 Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250 N on the surface ?

7.17 A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth ? Mass of the earth $= 6.0 \times 10^{24}$ kg; mean radius of the earth $= 6.4 \times 10^{6}$ m; $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.

7.18 The escape speed of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

7.19 A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg; mass of the earth $= 6.0 \times 10^{24}$ kg; radius of the earth $= 6.4 \times 10^{6}$ m; $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.

7.20 Two stars each of one solar mass ($= 2 \times 10^{30}$ kg) are approaching each other for a head on collision. When they are a distance 10^9 km, their speeds are negligible. What is the speed with which they collide ? The radius of each star is 10^4 km. Assume the stars to remain undistorted until they collide. (Use the known value of G).

7.21 Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid point of the line joining the centres of the spheres ? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable ?